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FUZZY AHP METHODOLOGY AND ITS SOLE APPLICATIONS

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ABSTRACT

Fuzzy AHP has been popular multi criteria decision making approach having varied application. Fuzzy AHP approach is assistance to the judgments with multiple preferences having fuzziness and uncertainty. As of now there are no much supportive surveys on FAHP application in multi criteria decisions problems. Research illustrates fuzzy AHP methodology and also registers few inimitable applications of fuzzy AHP.

Keywords: Multi criteria Decisions, Fuzziness, Chang's extent analysis, Fuzzy AHP.

INTRODUCTION

In real life, problems are with numerous constraints; here difficulty is to decide best among various constraints or alternatives. These constraints or alternatives are sometime conflicting in nature which increases complexity in decision making. Captivating precise decision to the problem considering all constraints is real difficult task. Decision making difficulty is aggregated when constraints are imprecise, vague, uncertain and fuzzy in nature. Methods used to solve such real life problems with numerous constraints are categorized into Multi criteria decision making (MCDM) methods, which are further classified into multi-attribute decision making (MADM) methods and multi objective decision making (MODM) methods (Hwang and Yoon, 1981). Decision made by human beings under numerous conflicting constraints is not reliable as human brain is able to consider and process only limited information at instant (Simpson 1996). To aid decision maker to solve real life problems and prioritized constraints, Thomas Saaty (1980) introduced AHP. The approach is based on pairwise comparisons between alternatives and depiction best possible alternative (Alptekin, 2012). AHP potency lies in its impartial, logically grading and also flexibility to integrate with various different techniques like Quality function deployment, Linear programming, fuzzy logic etc. The flexibility of AHP and ease of integrity with other methods gives advantage to user to extract maximum benefits out of method (Kubler et al., 2016)

AHP combined with fuzzy logic commonly known as Fuzzy AHP is practically popular method to deal with uncertainty and fuzziness and aid decision maker in complex problems with multiple conflicting criteria's (Kubler et al., 2016). This articles looks into application of Fuzzy AHP in various field by referring research papers published starting from 2003. The article is structured as follows, section two defines AHP and discuses pairwise comparison, section three presents fuzzy AHP and along Chang's extend analysis, section four details fuzzy AHP application in various fields. Finally section five summarizes and concludes studies with suggestion for further studies.

ANALYTICAL HIERARCHY PROCESS

AHP was introduced by Thomas Saaty (1980), is an effective tool for dealing with complex decision making, and may aid the decision maker to set priorities and make the best decision (Koul and Verma, 2015) (Rajesh and Malliga, 2013). By reducing complex decisions to a series of pairwise comparisons, and then synthesizing the results, the AHP helps to capture both subjective and objective aspects of a decision.

In addition, the AHP incorporates a useful technique for checking the consistency of the decision maker's evaluations, thus reducing the bias in the decision making process.

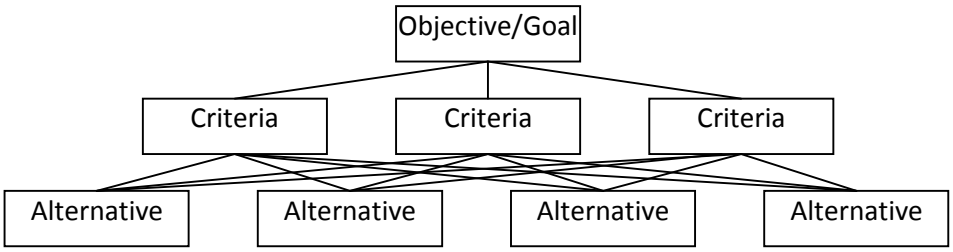

Fig. 1: Hierarchy structure for decision goal

The AHP considers a set of evaluation criteria, and a set of alternative options among which the best decision is to be made. Figure 1 shows hierarchy structure for decision goal using AHP approach. It is important to note that, since some of the criteria could be contrasting, it is not true in general that the best option is the one which optimizes each single criterion, rather the one which achieves the most suitable trade-off among the different criteria.

The AHP generates a weight for each evaluation criterion according to the decision maker's pairwise comparisons of the criteria. The higher the weight, the more important is corresponding criterion. Next, for a fixed criterion, the AHP assigns a score to each option according to the decision maker's pairwise comparisons of the options based on that criterion. The higher the score, better the performance of the criteria with respect to the considered criterion. Finally, the AHP combines the criteria weights and the options scores, thus determining a global score for each option, and a consequent ranking. The global score for a given option is a weighted sum of the scores it obtained with respect to all the criteria.

Analytical Hierarchy Process Model

AHP is summarized by following four Steps

Step 1: AHP uses several small sub problems to present complex decision problem. Thus first step is to decompose the decision problem into hierarchy with goal at the top, criteria, sub criteria at the levels, sub levels as shown in Figure 1.

Step 2: Construction of decision matrix based on Saaty's nine point scale which is indicated in Table 1. The decision matrix is pairwise comparison matrix to indicate relative importance of one criteria over other or one sub criteria over other at each level of hierarchy. If matrix formed is Anxn = [Aij], where the Aij entry represents the relative importance of decision maker `i ' with respect to decision maker `j '. That is, how much more important the ith

decision maker is relative to the jth decision maker. To illustrate this concept, a one may be asked how much more important is one decision maker compared to other in making decision. If Aij=1 then,

$$Aji = \frac{1}{Aij}$$

Table 1: Saaty's scale

Numerical Assessment	Linguistic meaning
1	Equal important
3	Moderate more important
5	Strong more important
7	Very strong important
9	Extreme more important
2,4,6,8	Intermediate values of importance

Step 3: Calculate priority weights of alternatives according to pairwise matrix. Normalization of pairwise matrix is done before calculating priority weights. Common method to normalized matrix Anxn formed earlier is by using following expression

$$Nij = \frac{Aij}{\sum_{i=1}^{n} Aij}$$

Priority vector Pi or weight vector is average the Nij.

$$Pi = \frac{\sum_{j=i}^{n} Nij}{J}$$

Step 4: AHP requires calculation of consistency ratio (CR), that indicates the consistency of decision makers judgment during evaluation process. For consistency Test of response is considered perfectly consistence , if matrix Aij has the following property:

$$Aij \times Pi = \lambda max \times Pi$$

where Pi is the eigen vector corresponding to the eigen value λ_{max}

Consistence index (CI) is given by

$$CI = \frac{\lambda max - n}{n - 1}$$

Finally, the consistency ratio (CR) is given by

$$CR = \frac{CI}{RI}$$

Table 2: Random consistency index (RI)

No.	1	2	3	4	5	6	7	8	9	10
RCI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Where RI is random consistency index obtain from Table 2 and depends upon dimension of decision matrix. The set of numbers in Table 2 is average random consistency index derived from sample of randomly generated reciprocal matrices using the scale 1/9,1/8..... 8,9 .If CR<

0.1, accept the pairwise comparison matrix. An inconsistency of 10 percent or less implies that the adjustment is small compared to the actual values of the eigenvector entries.

The concept of AHP and FAHP is mainly the same, and the difference of FAHP is that it analyses the numbers is term of fuzzy number but as for AHP the numbers analyzed are crisp numbers.

FUZZY ANALYTICAL HIERARCHY PROCESS

In real world there exists fuzzy knowledge which is unclear, vague, imprecise, inexact and probabilistic in nature. Human thinking and reasoning frequently involves information, which is inexact in nature. Human gives satisfactory answers which are probabilistic and not exact.

Zadeh introduced fuzzy set theory to cope with imprecision and uncertainty which is inherent to human judgement in decision making process through the use of linguistic term and degree of membership. Fuzzy set theory is extension of classical set theory, where elements have varying degree of membership (Amindoust et al, 2012). Classical set theory is based on 0 (False), 1 (True) logic, which is sometime insufficient to described human reasoning. Fuzzy logic uses whole interval between 0 and 1 to describe human reasoning. A major contribution of fuzzy set theory is its capability of representing vague data. Fuzzy set resembles human reasoning in its use of approximate information and uncertainty to generate decision. It was specially designed to mathematical represent uncertainty and vagueness and provide formalized tools for dealing with imprecision intrinsic of many problems.

A tilde "~" will be placed above e a symbol if the symbol shows a fuzzy set number. A Triangular Fuzzy Number (TFN) M is shown in Figure 2 as one illustration. A TFN is denoted simply as (a, b, c). The parameters a, b and c ($a \leq b \leq c$), respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event or set. The membership function of TFN is as follows:

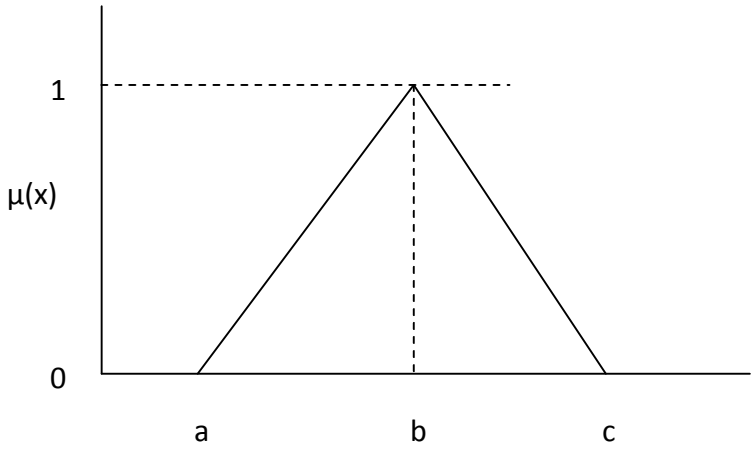

Fig. 2: Triangular fuzzy number

$$\mu_{\tilde{}}(x) = \begin{cases} c - x \end{cases}$$

While there are various operation on TFN's only the important operations are illustrated below. If (a1, b1, c1) and (a2, b2, c2) are two TFN's then addition, subtraction, multiplication division and inverse of TFN is as given below:

Addition: (a1, b1, c1) + (a2, b2, c2) = (a1+a2, b1+ b2, c1+ c2)

Subtraction: (a1, b1, c1) - (a2, b2, c2) = (a1-a2, b1- b2, c1- c2)

Multiplication: (a1, b1, c1) X (a2, b2, c2) = (a1a2, b1 b2, c1 c2)

Division: (a1, b1, c1) ÷ (a2, b2, c2) = (a1/a2, b1/b2, c1/ c2)

Inverse: (a1, b1, c1) = (1/ c1, 1/ b1, 1/ a1)

Fuzzy AHP (Chang's Extent Analysis

Chang's extent analysis (Agarwal and Singh) on fuzzy AHP depends on the degree of possibilities of each criterion. According to the responses on the question by decision maker, the corresponding triangular fuzzy values are determined for each criterion. And for a particular level on the hierarchy, the pair-wise comparison matrix is constructed. Sub totals are calculated for each row of the matrix and new set (a, b, c) is obtained, then in order to find the overall triangular fuzzy values for each criterion, ai/Σai, bi/Σbi,ci/Σci, (i=1,2,..., n) values are found and used as the latest Mi (ai, bi, ci)set for criterion Mi in the rest of the process. In the next step, membership functions are constructed for each criterion and intersections are determined by comparing each couple. In fuzzy logic approach, for each comparison the intersection point is found, and then the membership values of the point correspond to the weight of that point. This membership value can also be defined as the degree of possibility of the value. For a particular criterion, the minimum degree of possibility of the situations, where the value is greater than the others, is also the weight of this criterion before normalization. After obtaining the weights for each criterion, they are normalized and called the final importance degrees or weights for the hierarchy level. To apply the process depending on this hierarchy, according to the method of Extent analysis, each criterion is taken and extent analysis for each criterion is performed respectively.

According to the method of Chang's extent analysis each decision maker is taken and extent analysis for each goal of decision weight is performed respectively (Koul and Verma, 2011). Therefore, m extent analysis values for each decision maker can be obtained, with following signs:

$$M_{gi}^{1},\ M_{gi}^{2},\ M_{gi}^{3},\ \ldots\ldots\ M_{gi}^{n},\quad i = 1, 2, 3\ldots\ldots n.$$

Where, M_{gi}^{j}, (j = 1,2,3,.....m.) all are TFNs. The steps of Chang's extent analysis are as follows

The value if fuzzy synthetic extent with respect to the i^{th} decision maker is defined as

$$Si = \sum_{j=1}^{m} M_{gi}^{j} \ X \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$$

To obtain $\sum_{j=1}^{m} M_{gi}^{j}$, the fuzzy addition operation of m extent analysis values for particular matrix is performed is performed such as $\sum_{j=1}^{m} M_{gi}^{j} = \sum_{j=1}^{m} a1, \sum_{j=1}^{m} b1, \sum_{j=1}^{m} c1$

To obtain $\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}$, the fuzzy addition operation of M_{gi}^{1} , M_{gi}^{2} , M_{gi}^{3} , ... M_{gi}^{n} , i = 1, 2, 3.....n. is performed such as $\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j} = \sum_{i=1}^{n} ai, \sum_{i=1}^{n} bi, \sum_{i=1}^{n} ci$

And then inverse of the vector above is computed such as follows

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left[\frac{1}{\sum_{i=1}^{n} ai}, \frac{1}{\sum_{i=1}^{n} bi}, \frac{1}{\sum_{i=1}^{n} ci}\right]$$

Weights of decision makers (Dw) are determined at the end of process as fuzzy triangular number

FUZZY AHP APPLICATION

Numerous application of FAHP are uttered by different researchers from its introduction. FAHP is used to determine preference weighting of criteria's for decision maker by subjective perception (Shaverdi et al, 2012). This section presents sole application of FAHP based on collected articles from reputed Journals to guide researchers and practitioners to understand applicability of FAHP in various decision making domains.

Internet is one of basic tool of communication in market of product and service; internet provides platform to trade product and service to end users and also facilitate business transaction likes placing orders and payments. It becomes necessary for enterprise to carry E commerce activities by providing necessary services and information online. Growing E commerce websites made end user to evaluate success factors of E commerce websites (Hong and Liu, 2005). Evaluates success factor of E commerce website using fuzzy AHP approach, considering trust, system quality, content quality, online qualities, use as criteria and security, privacy, visual appearance, response time, page loading speed, 24 hrs availability, up to date, understand ability, timeliness, precision, tracking order status, account maintenance, payment alternative, information, transaction, the disabled service and FAQs as sub criterions.

Teaching performance evaluation is one of the main instruments to improve quality of teaching and also strengthen management of higher education institute (Do and Chen, 2013) Uses fuzzy AHP to identify teaching performance index along factors and sub factors weight calculation, based on weight frame work was developed to evaluate teaching performance. The paper used case application to demonstrate frame work application. The paper claims application of framework not only evaluates teaching performance but also highlights achievements of lectures and helps them to know improvement areas.

The paper demonstrate application of fuzzy AHP for comparing catering firms and select best firm providing most customer satisfaction. Paper uses interview approach with designed questionnaire to get data (Kahraman et al, 2004). Paper uses fuzzy AHP model with 32 listed criteria from product and management aspects for ERP selection problem and applies model to semiconductor and education industry, paper concludes their exist diversity in priorities between weights of criteria for two cases (Lein and Chan, 2007). Fuzzy AHP was used for selection of wastewater facilities at prefecture level. Criteria considered for pair wise comparison included land planning, environmental and techno economic consideration.

Generic developed model was applied in Envros prefecture at municipality of New Vissa (Anagnostopoulos et al, 2007).

Bridge construction projects are successful if appropriate bridge construction method is used. Paper uses triangular and trapezoidal fuzzy number with α concept to select best among Full span and pre cast launching method, advancing shoring method and balanced cantilever method. Fuzzy AHP determines weights of criteria and sub criteria. Criteria includes Quality, Cost, Safety, Duration, Shape and sub criteria includes durability, suitability, damage cost, construction cost, traffic conflict, site condition, constructability, weather condition, landscape and geometry environmental preservation (Pan,2008).Fuzzy AHP was applied to select best river considering vague data on efficient use of river system (Alias et al, 2009). Fuzzy AHP with chang's extent analysis was used for selecting shipping registry for existing fleet (Celik et al, 2009).

Considering 12 evaluation criteria's and bank experts opinion fuzzy AHP combined with fuzzy Delphi is used for evaluating lubricant regenerative technology (Hsu et al, 2010). Fuzzy AHP has proved to have potential to benefit manufacturing industry by minimizing negative effect while selection of lead free equipment (Tang and Lin 2010). Fuzzy AHP can be as one tool, that can help recruit manpower for organization (Agarwal and Singh). Banks is one of important sector that helps to built economy and investment, fuzzy AHP can be used by investors and decision makers for evaluating performance of banks (Akkoc and Vatansever, 2013). Service quality is driver for marketing and financial performance. Evaluating service satisfaction is multi criteria decision problem with qualitative and quantitative criteria and fuzzy AHP can be effective methodology for evaluation of service quality (Li, 2014). QFD integrated with fuzzy was used to select best supplier among ten different supplier (Patil et al, 2014). Fuzzy AHP is also used to select learning management system for WEBD considering 9 different criteria's and 10 different alternatives (Isik et al 2015)

Business unit prioritization can also be done using fuzzy AHP (Noori, 2015). Fuzzy AHP is also applied to prioritize low carbon technology to develop electricity (Promentilla et al, 2016). Considering 5 criteria's and 5 alternatives desalination system is selected using fuzzy AHP (Eusebio et al, 2016). Fuzzy AHP combined with GRA is used for selection of car considering qualitative and quantitative criteria. Car is selected from shortlisted five cars and short listing is done considering customer budget (Patil et al, 2017).

CONCLUSION

Human decision coupled with multi alternatives are usually based on experience judgment. Decision taken on experience may result selecting incorrect alternative resulting loss. So, scientific and methodological approach is required that will condense possibilities of selecting wrong decision. Fuzzy AHP is scientific approach that assists to take decision under fuzzy condition. It compares alternatives pairwise and select best among all. The paper shows fuzzy AHP methodology and also different distinctive applications of fuzzy AHP combined with other methods. Even after critics method is preferred by researcher to select best among numerous alternatives.

REFERENCES

[1] Aggarwal R, Singh S. AHP and Extent Fuzzy AHP Approach for Prioritization of Performance Measurement Attributes. World Academy of Science, Engineering and Technology 2013; 7(1): 43-48.

[2] Akkoc S, Vatansever K. Fuzzy performance evaluation with AHP and Topsis methods: evidence from turkish banking sector after the global financial crisis. Eurasian Journal of Business and Economics 2013; 6(11): 53-74.

[3] Alias MA, Hashim SZM, Samsudin S. Using fuzzy analytic hierarchy process for southern Johor river ranking. International Journal of Advance Soft Comput. Application 2009; 1(1): 62-76.

[4] Alptekin SE. A fuzzy decision support system for digital camera selection based on user preferences. Expert Systems with Applications 2012; 39(3)3037-3047.

[5] Amindoust A, Ahmed S, Saghafinia A, Bahreininejad A. Sustainable supplier selection: A ranking model based on fuzzy inference system. Applied Soft Computing 2012; 12(6): 1668-1677.

[6] Anagnostopoulos KP, Gratziou M, Vavatsikos AP. Using the fuzzy analytic hierarchy process for selecting waste water facilities at prefecture level. European Water 2007; 19(20): 15-24.

[7] Celik M, Ozok AF. Application of fuzzy extended AHP methodology on shipping registry selection: The case of Turkish maritime industry. Expert Systems with Applications 2009; 36(1): 190-198.

[8] Do QH, Chen JF. An evaluation of teaching performance: the fuzzy AHP and comprehensive evaluation approach. WSEAS Transactions on Information Science and Applications 2013; 10(1): 90-99.

[9] Eusebio RCP, Huelgas-Orbecido AP, Promentilla MAB. Optimal selection of desalination systems using fuzzy ahp and grey relational analysis. Chemical Engineering Transactions 2016; 52: 649-654.

[10] Hsu YL, Lee CH, Kreng VB. The application of Fuzzy Delphi Method and Fuzzy AHP in lubricant regenerative technology selection. Expert Systems with Applications 2010; 37(1): 419-425.

[11] Hwang CL, Yoon K. Multiple Attribute Decision Making: Methods and Applications, Springer-Verlag., New York, 1981.

[12] Isik AH, Ince M, Yigit T. A fuzzy AHP approach to select learning management system. International Journal of Computer Theory and Engineering 2015; 7(6): 499-502.

[13] Kahraman C, Cebeci U, Ruan D. Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey. International Journal of Production Economics 2004; 87(2): 171-184.

[14] Kong F, Liu H. Applying fuzzy analytic hierarchy process to evaluate success factors of e-commerce. International Journal of Information and Systems Sciences 2005; 1(3-4): 406-412.

[15] Koul S, Verma R. Dynamic vendor selection based on fuzzy AHP. Journal of Manufacturing Technology Management 2011; 22(8): 963-971.

[16] Kubler S, Robert J, Derigent W, Voisin A, Le Traon Y. A state-of the-art survey & testbed of fuzzy AHP (FAHP) applications. Expert Systems with Applications 2016; 65: 398-422.

[17] Li Y. Service Quality Measurement Using Fuzzy AHP-Based Approach. Journal of Computers 2014; 9(7): 1697-1703.

[18] Lien CT, Chan HL. A selection model for ERP system by applying fuzzy AHP approach. International Journal of the computer, the internet and management 2007; 15(3): 58-72.

[19] Noori B. Prioritizing strategic business units in the face of innovation performance: Combining fuzzy AHP and BSC. International Journal of Business and Management 2015; 3(1): 36-56.

[20] Pan NF. Fuzzy AHP approach for selecting the suitable bridge construction method. Automation in construction 2008; 17(8): 958-965.

[21] Patil AN, Bhale NGP, Raikar N, Prabhakaran M. Car Selection Using Hybrid Fuzzy AHP and Grey Relation Analysis Approach. International Journal of Performability Engineering 2017; 13(5): 569-576.

[22] Patil AN, Gaitonde VN, Dani MH. Supplier Selection using Fuzzy Integrated Approach. International Journal of Management Research and Reviews 2014; 4(6): 660-678.

[23] Promentilla MAB, Antonio MRS, Chuaunsu II RMM, Serra AJ de. A Calibrated Fuzzy AHP Approach to Derive Priorities in a Decision Model for Low Carbon Technologies. DLSU Research congress, 2016

[24] Rajesh G, Malliga P. Supplier selection based on AHP QFD methodology. Procedia Engineering 2013; 264(3); 1283-1292.

[25] Shaverdi M, Akbari M, Emamipour S. Using fuzzy multi criteria decision making approach for ranking the web browsers. International Journal of Economics and Management Science 2012; 1(8): 72-86.

[26] Simpson L. Do Decision Makers Know What They Prefer?: MAVT and ELECTRE II. The Journal of the Operational Research Society 1996; 47(7): 919-929.

[27] Tang YC, Lin TW. Application of the fuzzy analytic hierarchy process to the lead-free equipment selection decision. International Journal of Business and Systems Research 2010; 5(1): 35-56.

